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SUBJECT : MATHEMATICS **CLASS : XII**

CHAPTER: 1. RELATIONS AND FUNCTIONS

1. If $R_1\ and\ R_2$ are equivalence relations in a set A , show that $R_1 \cap R_2$ is also an equivalence relation.

2. Let R be the relation on set A of ordered pairs of positive integers defined by (x , y) R ((u , v) if and only if xv = yu. Show that R is an equivalence relation.

3.Let X = { 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 , 9 }.Let R_1 be a relation in X given by R_1 = {(x , y) : x – y is divisible by 3} and R_2 be another relation on X given by R_2 = {(x , y) : { x , y } is a subset of { 1 , 4 , 7} or { x , y } is a subset of { 2 , 5 , 8 } or { x , y } is a subset of { 3 ,6 ,9 }.Show that $R_1 = R_2$.

4. Let A = { 1 , 2 , 3 }. Then show that the number of relations containing (1 , 2) and (2 , 3) which are reflexive and transitive but not symmetric is four.

5. Show that the number of equivalence relations in the set { 1 , 2, 3 } containing (1 , 2) and (2 , 1) is two.

6. Consider a function f : [0 , $\frac{\pi}{2}$] → R given by f(x) = sinx and g : [0 , $\frac{\pi}{2}$] → R given by g(x) = cosx. Show that f and g are one one but f + g is not one one.

7. Show that the relation ‘ is a factor of ‘ on the set N of all natural numbers is reflexive and transitive but not symmetric.

8. Let A = Q x Q. Let ‘*’ be a binary operation on A defined by : (a,b) * (c,d) = (ac , ad+b) . Find i) Identity element of (A,*) & ii) the invertible element of (A,*) , if exists.

9. Let * be a binary operation on Z be defined as a * b = a + b – 15 for all a,b $\in$ Z , then

- i) Show that * is commutative and associative.
- ii) Find the identity element in (Z , *) .
- iii) Find the inverse of an element in (Z , *) .

10. Let f,g: R → R be defined as f(x) = $|\ x\ |$ and g(x) = [x] , where [x] denotes the greatest integer function less than or equals to x. Find fog ($\frac{5}{2}$) and gof (- $\sqrt{2}$).

11. If f : A → B and g: B → C are onto functions , then show that gof: A → C is also onto.

12. If f(x) = $\frac{x-1}{x+1}$, $(x\ \neq 1\ , -1)$, show that f o f^{-1} is an identity function.

13. If the function f : R → R is given by f(x) = $\frac{x+3}{2}$ and g : R → R is given by g(x) = 2x – 3 , find fog and gof. Is f^{-1} = g .

14. On R – {1} , a binary operation is defined * is defined as a * b = a + b – ab. Prove that * is commutative and associative. Find the identity element for *. Also prove that every element of R – {1} is invertible.

15. Given the functions f(x) = sinx and g(x) = cosx are one one in [0 , $\frac{\pi}{2}$]. Prove that f + g is not one one in [0 , $\frac{\pi}{2}$].

16. If f : R – {2} → R – {3} is defined by f(x) = $\frac{3x+1}{x-2}$, where R is the set of real numbers , then show that f is invertible and hence find the value of f^{-1}.

17. A binary operation * is defined on the set X = R – { - 1 } by x * y = x + y + xy , $\forall\, x , y \in X$. Check whether * is commutative and associative. Find its identity element and also find the inverse of each element of X.

(Ans. e = 0 $\in X$ is an identity element for X. Inverse of x $\in X\ is\ \frac{-x}{1+x}$)

18. If N denote the set of all natural numbers and R be the relation on N X N defined by (a , b) R (c , d) , if ad(b + c) = bc(a + d). Then show that R is an equivalence relation.

19. Let f:N→R be a function defined as f(x) = $4x^2+12x+15$ .Show that f:N→ s, where s is range of f ,is invertible. Find also the inverse of f.

20. Let S= { 1,2,3,4} and * be an operation on S defines by a*b = r , where r is the least non negative remainder when product is divided by 5 . prove that * is a binary operation on s.

21. Let f : R → R be the signum function defined as $f(x) = \begin{cases} 1\ if\ x > 0 \\ 0\ if\ x = 0 \\ -1\ if\ x < 0 \end{cases}$ and g:R→ R be the greatest integer function given by g(x)= [x], where [x] is is the greatest integer less than or equal to x. Does fog and gof coincide in [0,1]?

CHAPTER: 2. INVERSE TRIGONOMETRIC FUNCTIONS

1. Find the value of $tan^{-1}\left(\frac{x}{y}\right) - tan^{-1}\left(\frac{x-y}{x+y}\right)$.

2. If $a_1 , a_2 , a_3 , \ldots\ldots\ldots , a_n$ be an arithmetic progression with common difference d, then evaluate the following expression

$\tan[tan^{-1}\left(\frac{d}{1+ a_1.a_2}\right) + tan^{-1}\left(\frac{d}{1+ a_2.a_3}\right) + tan^{-1}\left(\frac{d}{1+ a_3.a_4}\right) + \cdots\ldots\ldots\ldots.. + tan^{-1}\left(\frac{d}{1+ a_{n-1}.a_n}\right)$].

3. Solve for x , if $tan^{-1} 2x + tan^{-1} 3x = \frac{\pi}{4}$.

4. Solve for x , $tan^{-1} (x + 1) + tan^{-1} (x - 1) = tan^{-1}\frac{8}{31}$

5. Prove that $sin^{-1}\frac{4}{5} + sin^{-1}\frac{5}{13} + sin^{-1}\frac{16}{65} = \frac{\pi}{2}$

6. Prove that $cos^{-1}\frac{12}{13} + sin^{-1}\frac{3}{5} = sin^{-1}\frac{56}{65}$

7. Solve for x: $\tan^{-1}(x - 1) + \tan^{-1} x + \tan^{-1}(x + 1) = \tan^{-1} 3x$

8. Prove that $\tan^{-1}\left(\frac{6x-8x^3}{1-12\,x^2}\right) - \tan^{-1}\left(\frac{4x}{1\,-\,4x^2}\right) = \tan^{-1} 2x\ ;\ |2x| < \frac{1}{\sqrt{3}}$

9. Prove that $2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{5\sqrt{2}}{7} + 2\tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

10. Show that $\cos\left(2\ \tan^{-1}\frac{1}{7}\right) = \sin\left(4\tan^{-1}\frac{1}{3}\right)$

11. Find the value of x satisfying the equation $\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right) + \frac{1}{2}\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{2\pi}{3} , x > 0$

12. Solve the equation $\tan^{-1}(\frac{2x}{1-x^2}) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{3}, x > 0$

13. Find the value of x , if $\sin^{-1} 6x + \sin^{-1} 6\sqrt{3}x = -\frac{\pi}{2}$

14. Prove that $2\sin^{-1}\frac{3}{5} - \tan^{-1}\frac{17}{31} = \frac{\pi}{4}$

15. Prove the following: $\cot^{-1}\left(\frac{xy+1}{x-y}\right) + \cot^{-1}\left(\frac{yz+1}{y-z}\right) + \cot^{-1}\left(\frac{zx+1}{z-x}\right) = 0, (0 < xy, yz, zx < 1)$

16. Show that $tan^{-1}1 + tan^{-1}2 + tan^{-1}3 = 2\left(tan^{-1}1 + tan^{-1}\frac{1}{2} + tan^{-1}\frac{1}{3}\right)$

17. Show that $\cot^{-1} 1 + \cot^{-1} 2 + \cot^{-1} 3 = \frac{\pi}{2}$

18. If $\sin[\cot^{-1}(x+1)] = \cos(\tan^{-1} x)$, then find the value of x.

19. If $(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$, then find the value of x.

20. Prove that $\sin^{-1}(\frac{4}{5}) + \sin^{-1}(\frac{5}{13}) + \sin^{-1}(\frac{16}{65}) = \frac{\pi}{2}$

21. Solve $\sin^{-1}\frac{5}{x} + \sin^{-1}\frac{12}{x} = \frac{\pi}{2}$

22. Solve $\sin^{-1}(1 - x) - 2\sin^{-1} x = \frac{\pi}{2}$

23. Solve $\sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{3}$

24. Solve $\cos^{-1}(\frac{a}{x}) - \cos^{-1}(\frac{b}{x}) = \cos^{-1}(\frac{1}{b}) - \cos^{-1}(\frac{1}{a})$, $|a| \leq 1, |b| \leq 1$

25. Solve $\sin^{-1} x + \sin^{-1}(1 - x) = \cos^{-1} x$

26. Solve $\cos(\tan^{-1} x) = \sin(\cot^{-1}\frac{3}{4})$

27. Prove that $2\tan^{-1}(\tan\frac{\alpha}{2}\tan(\frac{\pi}{4} - \frac{\beta}{2})) = \tan^{-1}(\frac{sin\alpha\, cos\beta}{sin\beta + cos\alpha})$

28.Prove that $cos^{-1}\left[\frac{cos\alpha + cos\beta}{1 + cos\alpha.cos\beta}\right] = 2\,tan^{-1}\left(tan\frac{\alpha}{2}\,tan\frac{\beta}{2}\right)$

29.Write into simplest form : $sin^{-1}\left[\sqrt{x}\sqrt{1-x^2} - x\sqrt{1-x}\right]$.

30.Solve the equation $\sin[2\,cos^{-1}(\cot(2\,tan^{-1}x))] = 0$.

CHAPTER: 3 & 4. MATRICES AND DETERMINANTS

1. Let A = $\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and f(x) = $x^2 - 4x + 7$. Show that f(A) = 0 and use this result to find A^5.

2. If A and B are square matrices of same order and B is a skew symmetric matrix , then show that A□BA is a skew symmetric matrix.

3. Using properties of determinants, prove that $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$

4. For what value of x , the matrix A is singular , if A = $\begin{bmatrix} 1+x & 7 \\ 3-x & 8 \end{bmatrix}$?

5. Using properties of determinants , prove that $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$

6. Show that the triangle ABC is an isosceles triangle if the determinant

$$\begin{vmatrix} 1 & 1 & 1 \\ 1+cosA & 1+cosB & 1+cosC \\ cos^2A+cosA & cos^2B+cosB & cos^2C+cosC \end{vmatrix} = 0$$

7. For a 3X3 matrix A, given that $|A| = 3$, then find $|adj(A)|$.

8. Use matrix multiplication to divide Rs30,000 in two parts such that the total annual interest at 9% on the first part and 11% on the second part amounts Rs3060.

9.Find the matrix A such that $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$

10. If A = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ then find BA and use this to solve the system of equations x – y = 3 , 2x + 3y + 4z = 17 and y + 2z = 7.

Prove the following

11. $$\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$$

12. $$\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$$

13. $$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (bc+ca+ab)^3$$

14. $$\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc\,(a+b+c)^3$$

15. Find the product AB , where A = $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve

the equations x – y + z = 4 , x – 2y – 2z = 9 and 2x + y + 3z = 1.

16. If a , b , c are positive and unequal , show that the following determinant is negative:

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

Using properties of determinants , solve the determinants for x:

17. $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$

18. $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$

19. Using elementary transformations, find the inverse of $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{vmatrix}$

20. For what value of k, the matrix $\begin{bmatrix} 2-k & 4 \\ -5 & 1 \end{bmatrix}$ is not invertible?

21. Using properties of determinants show that $\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$ = (x + y + z) $(z - x)^2$.

22. If A is a matrix of order 2 x 3 and B is a matrix of order 3 x 5 , then what is the order of matrix $(AB)^T$?

23. Show that $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix}$ = (a + b + c) ($a^2 + b^2 + c^2$).

24. A matrix of order 3 x 3 has determinant 6. What is the value of │ 3A│ ?

25. Find the matrix A satisfying the matrix equation $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

26.Let A = $\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$, then show that $A^2 - 4A + 7I = 0$. Using this result calculate A^3.

27. In a Legislative assembly election, a political party hired a public relation firm to promote its candidate in three ways; telephone, house calls and letters. The numbers of contacts of each type in three cities A, B & C are (500, 1000, and 5000), (3000, 1000, 10000) and (2000, 1500, 4000), respectively. The party paid Rs. 3700, Rs.7200, and Rs.4300 in cities A, B & C respectively. Find the costs per contact using matrix method. Keeping in mind the economic condition of the country, which way of promotion is better in your view?

28. for keeping Fit **X** people believes in morning walk, **Y** people believe in yoga and Z people join Gym. Total no of people are 70.further 20% 30% and 40% people are suffering from any disease who believe in morning walk, yoga and GYM respectively. Total no. of such people is 21. If morning walk cost Rs 0 Yoga cost Rs 500/month and GYM cost Rs 400/ month and total expenditure is Rs 23000.
(i) Formulate a matrix problem.
(ii) Calculate the no. of each type of people.
(iii) Why exercise is important for health?

CHAPTER: 5. CONTINUITY AND DIFFERENTIABILITY

1. Discuss the continuity of the function f(x) = $\begin{cases} \frac{x}{|x|}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$

2. Let f(x) = $\begin{cases} \frac{1-\cos 4x}{x^2} & if\ x < 0 \\ a & \text{if } x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & if\ x > 0 \end{cases}$

Determine the value of a , so that f(x) is continuous at x = 0.

3. If y = a (1 + cos θ) $and\ x = a\ (\ \theta - sin\theta\ ), then\ find\ \frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$

4. Discuss the continuity of the function f(x) = $\begin{cases} \frac{x}{|x|+2x^2}, & x \neq 0 \\ 2, & x = 0 \end{cases}$ at x = 0

5. If cosy = x cos(a + y) and cosa ≠ 1 , $then\ prove\ that\ \frac{dy}{dx} = \frac{cos^2\ (a+y)}{sina}$

6. $If\ x\sqrt{1+y} + y\sqrt{1+x} = 0\ and\ x \neq y, then\ prove\ that\ \frac{dy}{dx} = \frac{-1}{(1+x)^2}$

7. For what values of a and b , the function f defined as f(x) = $\begin{cases} 3ax + b, & \text{if } x < 1 \\ 10, & \text{if } x = 1 \\ 3ax - 3b, & \text{if } x > 1 \end{cases}$ is continuous at x = 1?

8. $If\ x^y + y^x = a^b, then\ find\ \frac{dy}{dx}$

9. Using Langrange's mean value theorem , find a point on the curve y = $\sqrt{x-2}$ defined on the interval [2 , 3] , where the tangent is parallel to the chord joining the end points of the curve.

10. If y = ($\cot^{-1} x$)^2 , $then\ show\ that\ (1 + x^2)^2.\frac{d^2y}{dx^2} + 2x\ (1 + x^2)\frac{dy}{dx}$ = 2.

11. If $(\ cosx\ )^y = (\ cosy\ )^x, then\ find\ \frac{dy}{dx}$.

12. For what value of k , is the function f(x) = $\begin{cases} \frac{1-\cos 4x}{8x^2}, & if\ x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ continuous at x = 0?

13. If f(x) and g(x) are two functions derivable in [a , b] such that f(a) = 4 , f(b) = 10 , g(a) = 1 and g(b) = 3 , then show that for a < c < b , $f^{\square}(c) = 3\,g^{\square}(c)$.

14. Verify the hypothesis and conclusion of Lagrange's mean value theorem for the function f(x) = $\frac{1}{4x-1}$, $1 \le x \le 4$.

15. Verify Rolle's theorem for the function f(x) = log$\left(\frac{x^2+ab}{(a+b)x}\right)$ $in\ [a,b], where\ 0 < a < b.$

16. If f(x) = $\begin{cases} \frac{cos^2x - sin^2x - 1}{\sqrt{x^2+1} - 1} & , x \neq 0 \\ a & , x = 0 \end{cases}$ is continuous at x = 0 , then find the value of a.

17. Find $\frac{dy}{dx}$ $, when\ y = \sqrt{a + \sqrt{a + \sqrt{a + x^2}}}\ , where\ a\ is\ a\ constant$

18. Differentiate $cos^{-1}\left[\frac{1 - x^2}{1 + x^2}\right]\ w.r.t.\ tan^{-1}\left[\frac{3x - x^3}{1 - 3x^2}\right]$

19. If x = sint , y = sinkt , then show that (1 - x^2) $\frac{d^2y}{dx^2} - x\frac{dy}{dx} + k^2y = 0.$

20. Show that the function f(x) = | x - 1| +| x + 1| ,
$\forall\ x \epsilon R , is\ not\ differentiable\ at\ the\ points\ x = -1\ and\ x = 1$

21. If y = $e^{m \sin^{-1} x}$, then show that (1 - x^2) $\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0.$

22. If f(x) = $\sqrt{x^2+1} , g(x) = \frac{x+1}{x^2+1}\ and\ h(x) = 2x - 3 , then\ find\ f^{\square}[h^{\square}\{g^{\square}(x)\}].$

23. Find the value of k for which

$$f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x} & , \text{ if } -1 \le x < 0 \\ \frac{2x+1}{2x-1} , & \text{ if } 0 \le x < 1 \end{cases}$$

is continuous at x=0.

Ans. $\boldsymbol{k = -\frac{1}{2}}$

24. Find the value of a for which the function f is defined as

$$f(x) = \begin{cases} a \sin\frac{\pi}{2}(x+1) & , \text{ if } x \le 0 \\ \frac{tanx - sinx}{x^3} & , \text{ if } x > 0 \end{cases}$$

is continuous at x=0.

Ans. $\boldsymbol{a = \frac{1}{2}}$

25. Find the relationship between a and b, so that the function f defined by

$$f(x) = \begin{cases} ax+1, & \text{if } x \le 3 \\ ax+b \quad , & \text{if } x > 3 \end{cases}$$

is continuous at x=3.

Ans. 3a -3b =2

26.Show that the function f(x) is defined by

$$f(x) = \begin{cases} \frac{sinx}{x} + \cos x, & \text{if } x > 0 \\ 2, & \text{if } x = 0 \\ \frac{4(1-\sqrt{(1-x)}}{x}, & \text{if } x < 0 \end{cases}$$

is continuous at x=0.

27. If f(x) defined by the following, is continuous at x=0, then find the values of a, b and c.

$$f(x) = \begin{cases} \frac{\sin(a+1)x+sinx}{x}, & \text{if } x < 0 \\ c\,, & \text{if } x = 0 \\ \frac{\sqrt{x+bx^2-\sqrt{x}}}{bx^{\frac{3}{2}}}\,, & \text{if } x > 0 \end{cases}$$

Ans. $\boldsymbol{a = -\frac{3}{2}, c = \frac{1}{2}, b\epsilon R - \{0\}}$